MATH-5A TEST 1 v1 (UNIT 1 sections 1.4-1.8, 2.1 , 3.4i and ii)
Fall 2024

100 points **NAME:**____________________

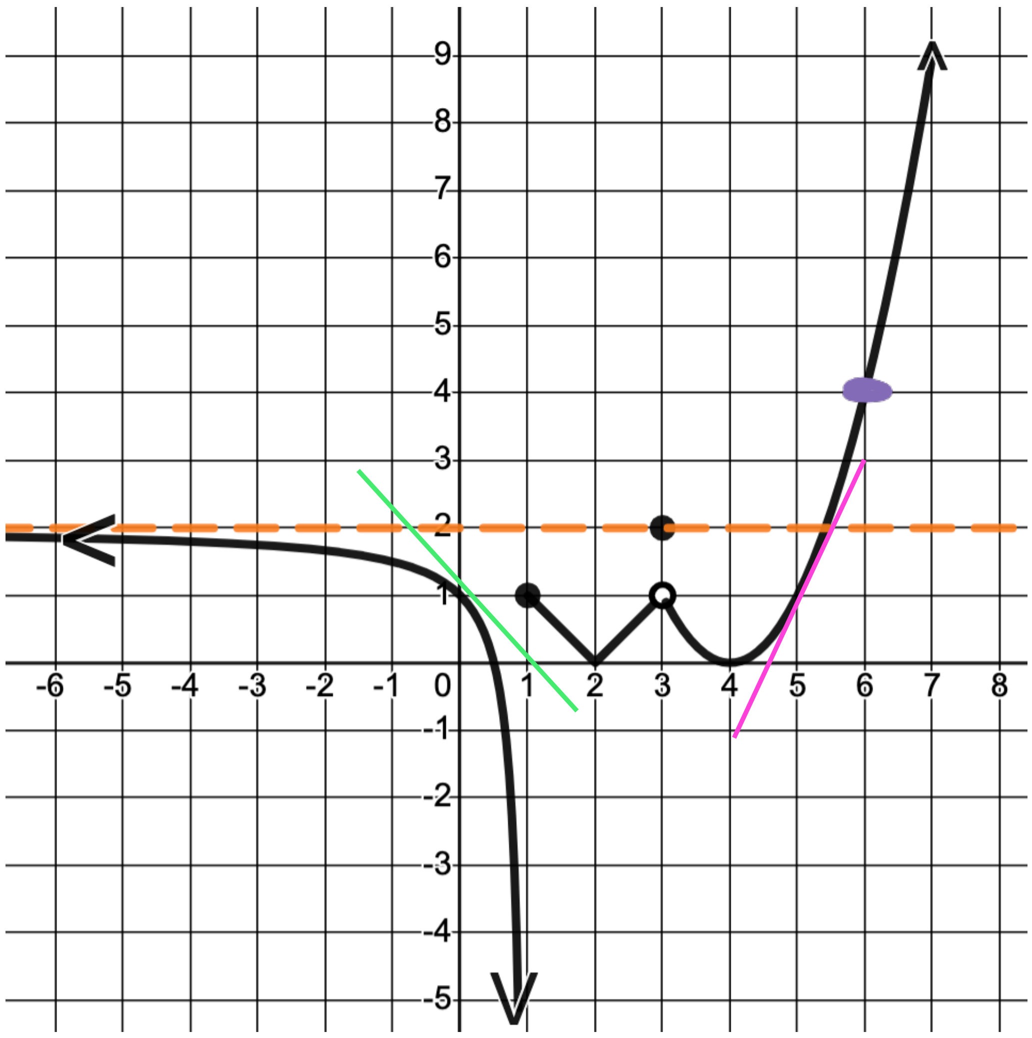
- **Detailed Instructions given in Canvas. Exact, simplified answers, good presentations with correct notation expected.**
- **Only methods learned in this class are allowed.**
- **All steps must be shown.**

FILL IN THE BLANKS WITH MOST APPROPRIATE ANSWER: (2 points each)

(1) One of the definitions of the derivative at $x = a$ is , $f'(a) = \lim_{x \to a}$ $\frac{f(x)-f(a)}{x-a}$.

(2) True or False: For any function f which Is continuous at x=a: $\lim_{x\to a} f(x) = f(a)$ True

(3) Given the graph of f(x) below, state the value of the following limits if they exist (if the limit is ∞ or $-\infty$ say so.) If the limit does not exist, answer DNE(1 points each)



(a) $\lim_{x\to 3^-} f(x)$ = 1.

(b) $\lim_{x\to 3^+} f(x)$ = 1.

(c) $\lim_{x\to 3} f(x)$ = 1.

(d) $f(3)$ = 2

(e) Is $f(x)$ continuous at x=3 no

(f) Estimate $f'(0)$ -1

(g) $\lim_{x\to 1^+} f(x)$ = 1.

(h) $\lim_{x\to 1^-} f(x)$ = $-\infty$

(i) $\lim_{x\to \infty} f(x)$ = ∞

(j) $\lim_{x\to -\infty} f(x)$ = 2

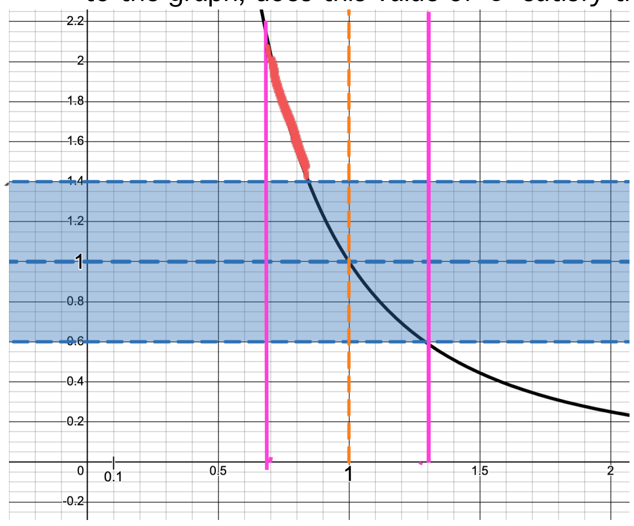
(k) find c so that $f(c) = 4$ 6

(l) find "a" so that $f'(a) \approx 2$ 5

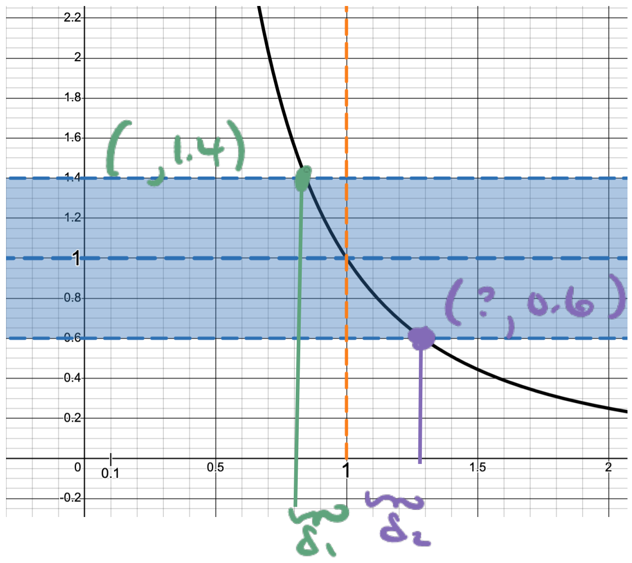
0.4

(4) *(10 points)*

(a) The graph below depicts $f(x) = 1/x^2$ with $\varepsilon = 0.2$, a=1, L=1 (as described in the definition of limit). On the grid, draw the vertical region corresponding to $|x-a| < \delta$ for $\delta = 0.3$. According to the graph, does this value of δ satisfy the definition for the given ε? no________



(b) Find a value for δ that satisfies the definiton of limit for this ε. Show work. The grid is for your use if desired.



$f(x)=1.4$

$\frac{1}{x^2} = 1.4$

$x^2 = \frac{1}{1.4} = \frac{10}{14} = \frac{5}{7}$

$x = \sqrt{5/7}$

$x \approx .845$

$\delta_1 \approx 1 - \sqrt{5/7}$

$\delta_1 \approx .155$

$f(x)=0.6$

$\frac{1}{x^2} = 0.6$

$x^2 = \frac{1}{0.6} = \frac{10}{6} = \frac{5}{3}$

$x = \sqrt{5/3}$

$x \approx 1.29$

$\delta_2 = \sqrt{\frac{5}{3}} - 1$

$\delta_2 \approx .291$

$\delta = \min\{\delta_1, \delta_2\}$

Answer: $\delta =$ $1-\sqrt{\frac{5}{2}} \approx .155$

(5) Evaluate the following limits if they exist (if the limit is ∞ or $-\infty$ say so.). No pr ... ou must your calculus techniques used in class. Making a table of values is not an acceptable technique.

(4 points each)

presentation as in class

(a) $\lim_{x\to 0} \frac{\sqrt{x+4}}{\sin x - 5}$ = $-2/5$

(b) $\lim_{x\to 1^+} \frac{-2+3x}{x-1}$ = ∞

" $\frac{-2+3}{0}$ " $\frac{\text{nonzero}}{0}$ $\frac{+}{+}$

" $\frac{0}{0}$ " (c) $\lim_{x\to 7} \frac{7-x}{3x^2-20x-7}$ = $-\frac{1}{22}$

$\lim_{x\to 7} \frac{7-x}{3x^2-20x-7} = \lim_{x\to 7} \frac{7-x}{(x-7)(3x+1)}$

$= \lim_{x\to 7} \frac{-1}{3x+1}$

$= \frac{-1}{22}$

" $\frac{0}{0}$ " (d) $\lim_{x\to 0^-} \frac{x}{|x|}$ = -1

$x\to 0^- \Rightarrow x<0$ so $|x| = -x$

$\lim_{x\to 0^-} \frac{x}{-x} = \lim_{x\to 0} (-1) = -1$

x^2-4

" $\frac{0}{0}$ " (e) $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x}$ = $\frac{1}{2\sqrt{3}}$

" $\frac{\infty}{\infty}$ " (f) $\lim_{x\to -\infty} \frac{2-3x}{\sqrt{4-x^2}}$ = 3

(Show details)

$\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x} \cdot \frac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}}$

$= \lim_{x\to 0} \frac{3+x-3}{x(\sqrt{3+x}+\sqrt{3})}$

$= \lim_{x\to 0} \frac{x}{x(\sqrt{3+x}+\sqrt{3})}$

$= \lim_{x\to 0} \frac{1}{\sqrt{3+x}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$

$\lim_{x\to -\infty} \frac{2-3x}{\sqrt{\cancel{4}-\cancel{x^2}}\,x^2-4} = \lim_{x\to -\infty} \frac{2-3x}{|x|\sqrt{x^2-4}}$

$x\to -\infty \Rightarrow x<0$ so $|x| = -x$

$= \lim_{x\to -\infty} \frac{2-3x}{-x\sqrt{1-4/x^2}} \cdot \frac{1/x}{1/x}$

$= \lim_{x\to -\infty} \frac{\frac{2}{x}^{\to 0}-3}{-\sqrt{1-4/x^2}^{\to 0}} = \frac{-3}{-1} = 3$

(6) If f(x)= $\begin{cases} \sqrt{7+x} & \text{if } x \le 2 \\ x^2+3 & \text{if } x > 2 \end{cases}$ find (8 points)

$\lim_{x\to 2^-} f(x)=$ __3__ $\lim_{x\to 2^+} f(x)=$ __7__ $\lim_{x\to 2} f(x)=$ __DNE__

$= \lim_{x\to 2^-} \sqrt{7+x}$

$= \sqrt{9} = 3$

$= \lim_{x\to 2^+} (x^2+3) = 7$

Is $f(x)$ continuous at x=2? __no__

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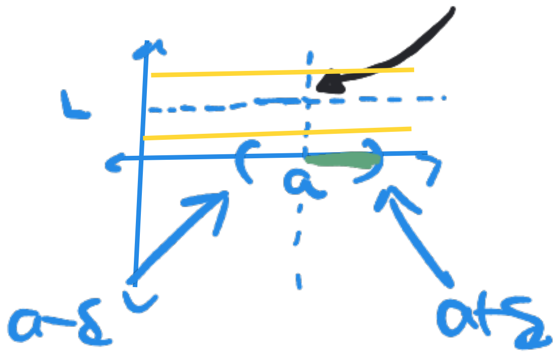
(7) Give the <u>formal/rigorous</u> definition for $\lim_{x\to a^+} f(x) = L$ (4 points)



Given any $\varepsilon > 0$ there is a $\delta > 0$ such that if $a < x < a+\delta$ then $|f(x)-L| < \varepsilon$

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(8) For what values of x are the following functions continuous? Show work. (4 points each)

a) f(x) = $\dfrac{x^2-1}{2\cos x + \sqrt{2}}$

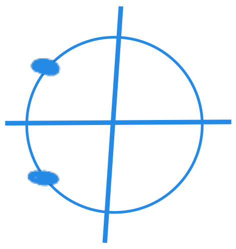
Conts. on domain

denom $\neq 0$

$2\cos x + \sqrt{2} \neq 0$

$\cos x \neq -\sqrt{2}/2$

$\frac{3\pi}{4}$



$5\pi/4$

f(x) is conts for all reel x except

$x = \frac{3\pi}{4} + 2\pi k, \frac{5\pi}{4} + 2\pi k$

k integer

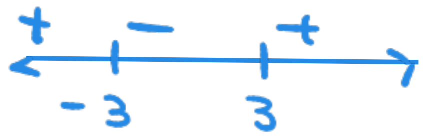
b) f(x) = $\sqrt{x^2-9}$

Conts. on domain

radicand ≥ 0

$x^2 - 9 \geq 0$

$(x-3)(x+3) \geq 0$



f(x) conts. on

$(-\infty, -3] \cup [3, \infty)$

(9) The table shows the position of a cyclist. (10 points)

t (seconds)	0	1	2	3	4	5
s (feet)	0	2.5	10.0	22.5	40.0	62.5

(Express answers using appropriate units.)

Estimate the instantaneous velocity of the cyclist at t=1 as accurately as possible using two different techniques, (a) using the numerical data in the table, and (b) using the given graph, graphing the corresponding tangent line and using the graph of the line . Explain what you are doing.

(a) Using numerical data.

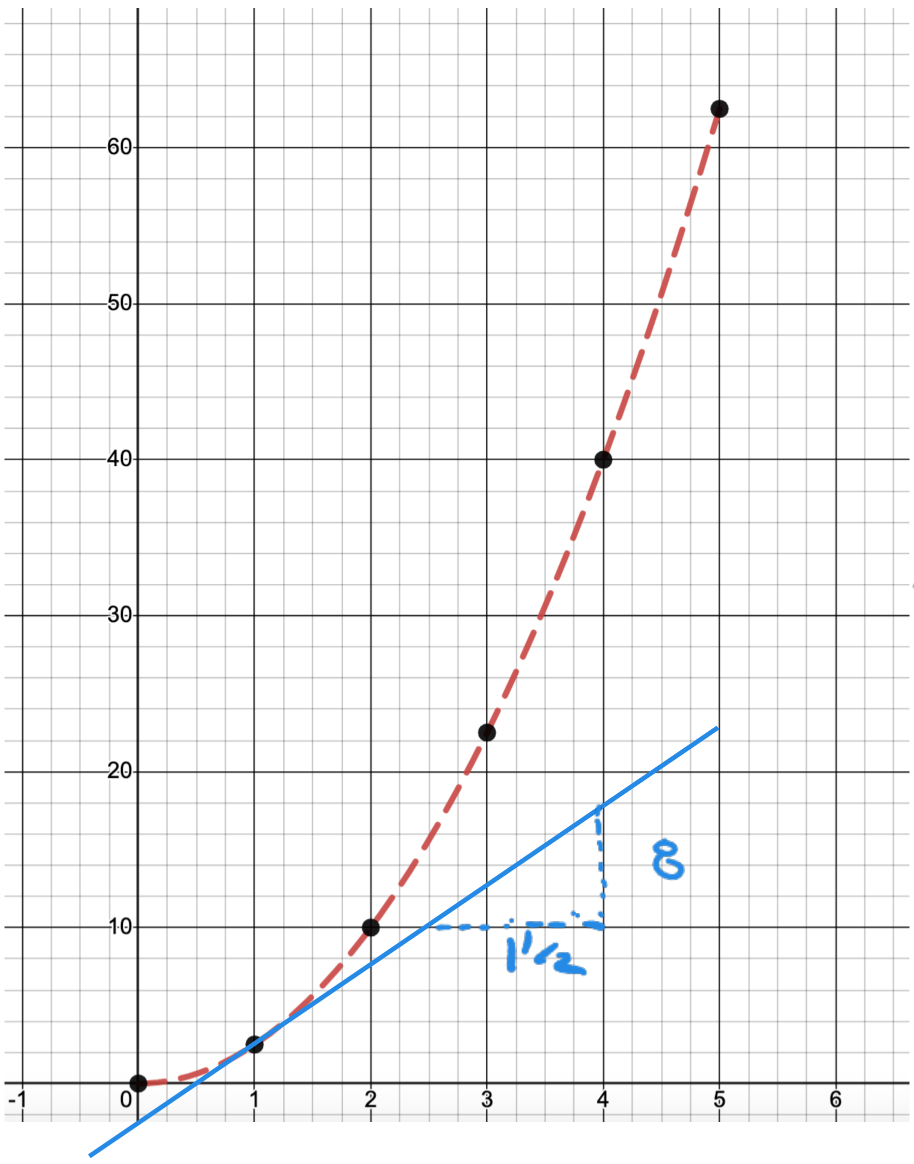
Ave. Velocity on $[0,1]$ $\frac{s(1)-s(0)}{1-0} = \frac{2.5-0}{1} = 2.5$ ft/sec

Ave. Velocity on $[1,2]$ $\frac{s(2)-s(1)}{2-1} = \frac{10-2.5}{1} = 7.5$ ft/sec

Take average
Inst velocity at $t=1$ $\approx \frac{2.5+7.5}{2}$

≈ 5 ft/sec

(b) Using the graph



Draw tangent line, Find slope from your graph

$m \approx \frac{8}{3/2} = \frac{16}{3} = 5\frac{1}{3}$ ft/sec

(10) Given $f(x)=x^2-2$ (20 points)

a) Use an appropriate form of the definition of the derivative to compute $f'(a)$.

$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

$=\lim_{x\to a}\frac{x^2-2-(a^2-2)}{x-a}$

$=\lim_{x\to a}\frac{x^2-a^2}{x-a}$

$=\lim_{x\to a}(x+a)$

$=2a$

OR

$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

$=\lim_{h\to 0}\frac{(a+h)^2-2-(a^2-2)}{h}$

$=\lim_{h\to 0}\frac{a^2+2ah+h^2-2-a^2+2}{h}$

$=\lim_{h\to 0}\frac{2ah+h^2}{h}$

$=\lim_{h\to 0}(2a+h)=2a$

ANS

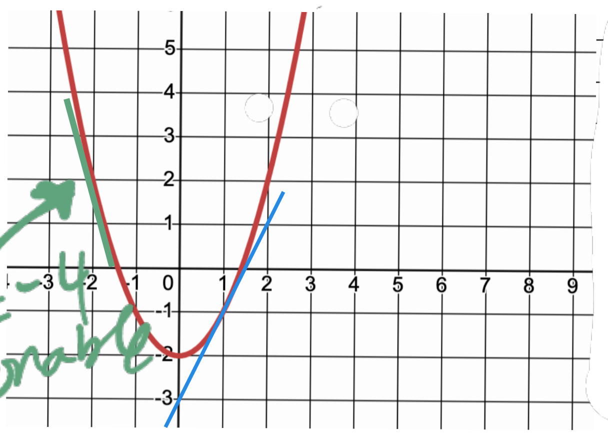
$f'(a)=2a$

b) Use the results of part (a) to find the slope of the tangent line at x = 0 and at x=2. $f'(2)=4$

slope at x= 0 __0____ slope at x= 1 __2____

c) Sketch a graph of f(x) and sketch the tangent line at x= 1 The slope in part (b) should be reasonable according to your graph.

Accepted both answers



$f'(-2)=-4$ reasonable

d) Find the equation of the tangent line at x=

$x=1$
$f(1)=-1$
$m=f'(1)=2$

$y+1=2(x-1)$
$y=2x-3$

$x=2$ $(2,2)$ $f'(2)=4$
$y-2=4(x-2)$
$y=4x-6$

e) Using part (a), find a value for a such that $f'(a)=-4$ This answer should be reasonable according to your graph.

$2a=-4$
$a=-2$